MTH 310 HW 6 Solutions

March 4, 2016

Section 3.3, Problem 35f

Prove $\mathbb{Z}_4$ x $\mathbb{Z}_4$ is not isomorphic to $\mathbb{Z}_{16}$.
Answer. Note that if they were isomorphic, then there would be an isomorphism $f : \mathbb{Z}_{16} \to \mathbb{Z}_4$ x $\mathbb{Z}_4$. But this implies that, since all isomorphisms send 0 to 0, $f(0) = 0$. But since $a+a+a+a = 0$ for all $a \in \mathbb{Z}_4$, if $f(1) = (a, b) \in \mathbb{Z}_4$ x $\mathbb{Z}_4$, we have $f(0) = 0 = (a+a+a+a, b+b+b+b) = (a, b)+(a, b)+(a, b)+(a, b) = f(1)+f(1)+f(1)+f(1) = f(1+1+1+1) = f(4)$. Therefore since $0 \neq 4$ in $\mathbb{Z}_{16}$, f is not injective. (Even more easily, we can note that f(1) must be the identity (1,1) in $\mathbb{Z}_4$ x $\mathbb{Z}_4$ since f is a homomorphism of rings with identity.)

Section 3.3, Problem 38a

Prove if $f : \mathbb{F} \to R$ is a homomorphism from a field to a ring, if $f(c) = 0$ for some nonzero $c \in \mathbb{F}$, then f is the zero map.
Answer. Assume there is some nonzero element $c \in \mathbb{F}$ for which $f(c) = 0$. Let $x \in \mathbb{F}$. Then as c is nonzero, there is some c^{-1} so $cc^{-1} = 1$, so $f(x) = f(1x) = f(cc^{-1}x) = f(c)f(c^{-1}x) = 0f(c^{-1}x) = 0$.

Section 4.1, Problem 4

Find polynomials $f(x), g(x), h(x), p(x)$ so that $\deg(f(x)+g(x)) = \max(\deg(f(x), \deg(g(x)))$ and $\deg(h(x) + p(x)) < \max(\deg(h(x), \deg(p(x)))$.
Answer. The polynomials $f(x) = 2x^2, g(x) = h(x) = x^2 + x$, and $p(x) = -x^2$ satisfy this. Answers may vary.